Pergamon

Journal of Structural Geology, Vol. 17, No. 7, pp. 1065 to 1070, 1995

0191-8141/95 $9.50+0.00

0191-8141(95)00003-8

Brevia

SHORT NOTES

Fault-slip calculation from separations

EIZO YAMADA and KEIICHI SAKAGUCHI

Geological Survey of Japan, 1-1-3 Higashi Tsukuba, 305 Japan

(*Received* 3 *May* 1994; *accepted in revised form* 30 *November* 1994)

Abstract—A computer program to calculate the slip vector of a fault from either: (1) the separations of two marker planes, or (2) the separation of a marker plane and the slip direction of the fault, measured at an outcrop, was revised in order to calculate the error range of the slip vector under given error ranges of the measured separations and directions. In some cases a small error range in the original data gives rise to large error ranges in the calculated slip direction and magnitude, and therefore it is very important to accurately assess the error range of data. The use of the program to solve some structural problems is also discussed.

INTRODUCTION

Graphical methods to calculate fault slip vectors from separations of two marker planes on a map have been described in textbooks on structural geology (e.g. Billings 1954, Ragan 1973). Yamada (1980) described a computer program to calculate the slip vectors from either: (1) the separations of two marker planes (e.g. a bedding plane and an older fault plane), or (2) the separation of a marker plane and the slip direction of the fault, measured at an outcrop.

These methods are applicable only if the fault and the marker planes approximate to perfect geometrical planes and if both walls of the fault have slipped as rigid bodies without rotation. However, in reality, faults and marker planes are not perfect planes, and both walls of a fault show strains. In addition, certain amounts of error will accompany measurement of data at an outcrop. These intrinsic and measurement errors, in some cases, completely jeopardize the reliability of calculated results. Therefore, the computer program (Yamada 1980) is revised to assess the influence of these errors on the calculated results. The applicability of the revised program to solve some of the structural problems is also discussed.

FIELD DATA ACQUISITION

At first, some terms used in this paper are defined. The fault slip vector direction is reversed depending on whether it is measured with respect to the hanging wall or the footwall. In this paper, the slip vector of the footwall block is adopted (i.e. the slip inscribed on the hanging wall). When the fault is vertical, the southern block is defined as the footwall block and when the fault is vertical and strikes N–S, the west block is defined as the footwall block for convenience. The slip vector is defined as positive upward. When the slip vector is horizontal, the easterly direction is defined as positive and when the slip vector is horizontal and north–south, the north is defined as positive.

The rake is defined as the angle between the slip vector and the northerly direction (when the strike is east–west, the east direction) of strike of the fault, and the negative upward rotation and positive downward rotation (Fig. 1). When the fault is horizontal, the north is defined as zero, the eastward rotation is positive and westward rotation is negative.

The separation vector reverses depending upon whether it is measured from the hanging wall block or the footwall block. Here, it is measured from the hanging wall block to the footwall block, and the sign of the vector is defined similarly to the sign of the slip vector.

At an outcrop, the dips and strikes of the outcrop plane, of a fault plane, and of marker planes are measured. The separations of the marker planes are measured along the line of intersection of the fault plane and the outcrop plane (Fig. 2). The rake of striation direction is also measured if possible. On the basis of these data, the fault slip is calculated from either: (1) the separations of two marker planes, or (2) the separation of a marker plane and the slip direction of the fault, by simple three-dimensional geometric calculations as follows.

FAULT-SLIP CALCULATION

In calculations of slip vectors, a rectangular coordinate system (Fig. 2) is used. In the following, a fault plane, a marker plane in the hanging wall and in the footwall blocks, another marker plane in the hanging wall and in the footwall blocks, and an outcrop plane are

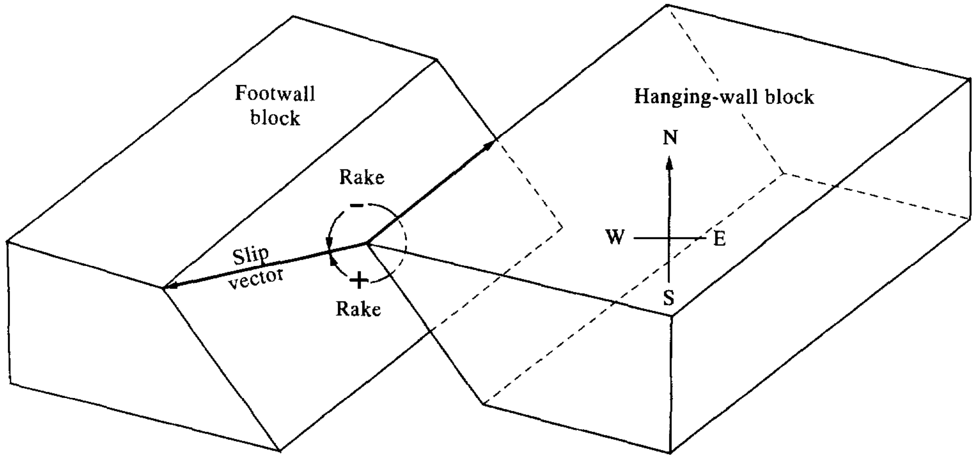

Fig. 1. Definitions of the slip vector and rake. The slip vector adopted is that of the footwall block. Rake is measured from the northerly strike direction (zero), and negative in upward rotation and positive in downward rotation.

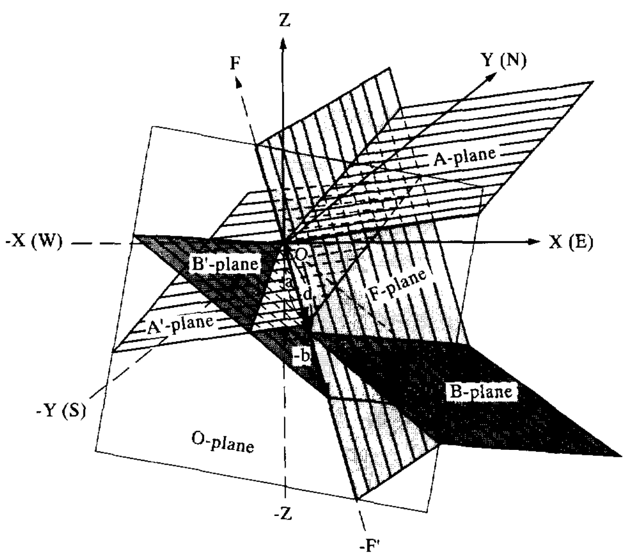

Fig. 2. Relationships between X-, Y- and Z-coordinate axes and the north(N), the south(S), the east(E) and the west(W). The origin(O) of coordinates is taken at the intersecting point of fault plane (F-plane), outcrop plane (O-plane) and a marker plane in the hanging wall block(A-plane). The upward direction along the outcrop fault trace (F, -F') is taken positive. The separation vectors (**-a**, **-b** and **-d**) of marker planes (A-plane to A'-plane, B-plane to B'-plane and A-plane to B-plane, respectively) are shown by thick arrows.

abbreviated as F-plane, A-plane and A'-plane, B-plane and B'-plane, and O-plane, respectively.

Firstly, poles to F-plane, A-plane, B-plane and O-plane are converted to the direction cosines (l_i, m_i, n_i), (l_j, m_j, n_j), (l_k, m_k, n_k) and (l_o, m_o, n_o), respectively.

Then the formulae for F-plane, O-plane and A-plane can be expressed as follows:

$$\text{F-plane, } l_i x + m_i y + n_i z = 0, \tag{1}$$

$$\text{O-plane, } l_o x + m_o y + n_o z = 0, \tag{2}$$

$$\text{A-plane, } l_j x + m_j y + n_j z = 0. \tag{3}$$

Direction cosines (l_p, m_p, n_p) of the line of intersection of F-plane and O-plane are obtained from (1) and (2),

$$l_p = \frac{m_i n_o - n_i m_o}{X},$$

$$m_p = \frac{n_i l_o - l_i n_o}{X}, \tag{4}$$

$$n_p = \frac{l_i m_o - m_i l_o}{X}.$$

Here, X denotes

$$\sqrt{(m_i n_o - n_i m_o)^2 + (n_i l_o - l_i n_o)^2 + (l_i m_o - m_i l_o)^2}.$$

If the separations from A-plane to A'-plane, B-plane to B'-plane, and A-plane to B-plane measured along the line of intersection of F-plane and O-plane are expressed respectively as a, b and d, the formulae for A'-plane, B-plane and B'-plane can be expressed as follows,

A'-plane,

$$l_j\,(x - al_p) + m_j(y - am_p) + n_j(z - an_p) = 0, \tag{5}$$

B-plane,

$$l_k(x - dl_p) + m_k(y - dm_p) + n_k(z - dn_p) = 0, \tag{6}$$

B'-plane,

$$l_k[x - (d + b)\, l_p] + m_k[y - (d + b)m_p] + n_k[z - (d + b)n_p] = 0. \tag{7}$$

The intersection point (x_1, y_1, z_1) of A-plane, B-plane and F-plane is calculated by solving (1), (3) and (6),

$$x_1 = \frac{-d(l_k l_p + m_k m_p + n_k n_p)\,(m_j n_i - n_j m_i)}{D},$$

$$y_1 = \frac{-d(l_k l_p + m_k m_p + n_k n_p)\,(n_j l_i - l_j n_i)}{D},$$

$$z_1 = \frac{-d(l_k l_p + m_k m_p + n_k n_p)\,(l_j m_i - m_j l_i)}{D}.$$

Similarly the intersecting point (x_2, y_2, z_2) of A'-plane, B'-plane and F-plane is calculated by solving (1), (5) and (7),

$$x_2 = \frac{a(l_j l_p + m_j m_p + n_j n_p)\,(m_k n_i - n_k m_i) - (d + b)(l_k l_p + m_k m_p + n_k n_p)(m_j n_i - n_j m_i)}{D},$$

$$y_2 = \frac{a(l_jl_p + m_jm_p + n_jn_p)(n_kl_i - l_kn_i) - (d + b)(l_kl_p + m_km_p + n_kn_p)(n_jl_i - l_jn_i)}{D},$$

$$z_2 = \frac{a(l_jl_p + m_jm_p + n_jn_p)(l_km_i - m_kl_i) - (d + b)(l_kl_p + m_km_p + n_kn_p)(l_jm_i - m_jl_i)}{D}.$$

Here, D denotes $\begin{vmatrix} l_j, m_j, n_j \\ l_k, m_k, n_k \\ l_i, m_i, n_i \end{vmatrix}$.

The magnitude (s) of slip vector of the fault is,

$$s = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}, \quad (8)$$

and the direction cosines (l,m,n) of the slip vector of the fault are,

$$l = \frac{x_2 - x_1}{s},\ m = \frac{y_2 - y_1}{s},\ n = \frac{z_2 - z_1}{s}.$$

Actually, d disappears in (8) and therefore it is not necessary to measure it to obtain the slip vector.

If the direction of the slip vector is known, the measured slip direction rake is converted to the direction cosines (l, m, n). Then, the magnitude of the slip vector is calculated by finding the distance from the origin to the point (x_3, y_3, z_3), where the line passing through the origin and extending in the direction of the slip vector meets A′-plane.

Hence, by solving for x, y, z from

$$\frac{x}{y} = \frac{l}{m},\quad \frac{y}{z} = \frac{m}{n},\quad \text{and (5)},$$

$$s = \sqrt{x_3^2 + y_3^2 + z_3^2}.$$

Thus, the calculation is very simple but during computation care must be taken to select an appropriate set of direction cosines out of the two sets with opposite sense.

ERROR RANGE CALCULATION

In this revision of the program the original data are considered to have certain errors. Slip direction rakes, dips and strikes are varied by several degrees and several percent is either added to or subtracted from the original separation data. Then, the calculations are carried out on all the combinations of possible errors. In the calculation of the error range of the slip vector from the separations of two marker planes, those combinations for which the slip direction makes the maximum angular deviation on either side of the slip direction calculated with the original data, are determined. In the calculation of slip vectors from the separation of a marker plane and the direction of fault-slip, those combinations for which the magnitude of slip vector shows the maximum or the minimum are determined. In many outcrops the sense of slip is difficult to determine and therefore the sense of slip is allowed to reverse during the calculations. The results are printed in tables and also displayed graphically.

Broadly, the calculation procedure is very simple but in detail several points are worthy of comment. Under some error conditions, the calculated slip vectors on some fault planes are directed toward both sides of the outcrop fault trace (F–F′ of Fig. 2); that is, toward the actually existing fault surface and toward the imaginary fault surface extended out of the outcrop surface. In such cases, the error ranges are calculated separately on either side, as there may be a gap between the two ranges. When a discontinuity occurs, such as when the sign of the slip or the separation vectors, or of rakes changes, special consideration is needed. If the dip is low, then the possible error of strike measurement may be large. In the program, error ranges of the strike angle are assumed inversely proportional to the size of the dip angle, when the dip is less than 5°.

APPLICABILITY TO SOME STRUCTURAL PROBLEMS

The error ranges of the slip vectors calculated from the separations of two marker planes vary widely depending on the data even if the same error ranges are assumed for all of the data. When the trace directions of two marker planes lie close to each other on the fault surface, small errors could reverse the calculated slip direction (Fig. 3e). The calculated slip vector may be a cumulative one with incremental slips in different directions. If the calculated slip is small, however, it has probably moved under roughly a single stress field. In such cases, if the direction of striae or of slip obtained by other means concurs with the direction of slip calculated from separations it may be safely concluded that these directions have been derived from the same stress field. If the complete sets of principal stress directions and stress difference ratios (R) which operated in an area are known from striation data analyses, the calculated slip vectors must theoretically be explained in terms of one or of combinations of two or more of the stress sets. Calculations on data from south Izu proved that, if appropriate error ranges are assumed for the data, the calculated slip vectors can be explained mostly in terms of one and rarely of combinations of the paleostress fields estimated from the striation data by Yamada & Sakaguchi (1991). Some of the data and results of calculations are shown in Table 1 and in Fig. 3. An incompatibility between the calculated slip directions and directions of maximum shear stress corresponding to calculated palaeostresses may occur, however, in three possibilities. Firstly, a different stress field could have existed locally or regionally from those so far considered. Secondly, local anomalies could have existed and the fault slipped in a different direction from the maximum shear direction, or the fault and strata have locally rotated. Thirdly, the data errors are larger than assumed.

The magnitude of the slip vector calculated from the separation of a marker plane and the striation data, postulating that the fault slipped in the striation direction with error ranges, also varies widely depending on

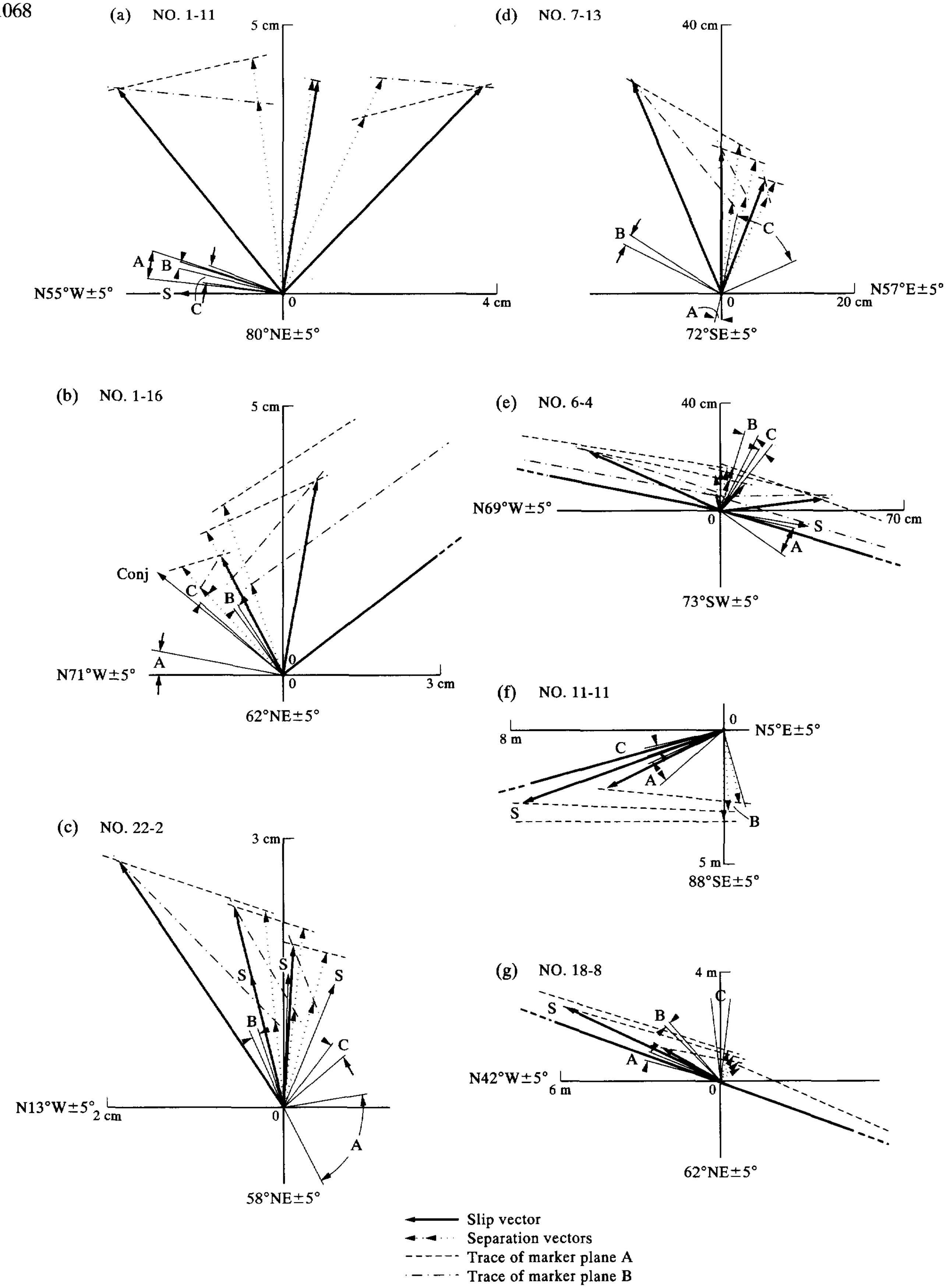

Fig. 3. The slip vector calculated from the original data and those showing maximum angular deviations from the original are plotted on the hanging walls of fault. The horizontal axis is the strike line and the vertical axis is the dip direction of faults. Fault numbers correspond to those in Table 1. (a)–(e) Slip vectors calculated from the separations of two marker planes. (f)–(g) Slip vectors calculated from the separation of a marker plane and the direction of striae. The assumed error ranges are ±10% for separations and ±5° for angular data. The corresponding separation vectors and the traces of marker planes are also plotted. When the error ranges of calculated directions of slip vector on either side of the outcrop trace have a gap between them, the slip vectors with maximum angular deviations are separately plotted on either side [i.e. (e) & (g)]. The directions of striations (*S*) and of conjugate slip (Conj), and the ranges of maximum shear stress direction obtained from the three estimated palaeostress tensors (A, B, C) after Yamada & Sakaguchi (1991) are also shown for comparison.

Table 1. Some examples of data and results of calculations. Fault numbers correspond to those in Fig. 3. For faults 11–11 and 18–8 calculations are from the separation of a marker plane and the slip direction (striae), and the others are for calculations from the separations of two marker planes. The assumed error ranges are ±10% for separations and ±5° for angular data. When the slip directions are on both sides of the outcrop trace in the fault plane, their error ranges are calculated separately on either side and are shown separately in the upper and the lower rows. Fault 18–8 requires a reversal of slip direction under the error ranges. The reversed striae rake is shown in parentheses

	Fault	Marker plane A		Marker plane B		Outcrop	Striae	Slip magnitude	(±10%, ±5°)	Slip direction rake	(±10%, ±5°)
Fault No.	Strike, dip	Sep. (cm)	Strike, dip	Sep (cm)	Strike, dip	Strike, dip	rake (°)	Value (cm)	Range (cm)	Value (°)	Range (°)
1–11	N55W, 80NE	N4	N63E, 20NW	N4	N69E, 10SE	N6W, 90	0?	4	5, 4	−99	−134, −87
									4, 5		−114, −51
1–16	N71W, 62NE	N3	N54E, 24NW	N2	N25W, 72SW	N60E, 84SE	−39*	4	70, 3	−100	−142, −63
6–4	N69W, 73SW	N15	N33W, 22NE	N6	N69W, 65NE	N35E, 90	170	41	55, 720	−172	−24, −11
									310, 15		164, 223
7–13	N57E, 72SE	N20	N22W, 22NE	N15	N67W, 74NE	N6W, 85SW	−76	22	34, 18	−90	−114, −68
22–2	N13W, 58NE	N110	N65W, 80SW	N200	N44W, 35SW	N82E, 80NW	−92	230	180, 330	−76	−93, −56
							−112				
11–11	N5E, 88SE	(R>300)	N13E, 12NW			N40W, 90	160	810	510, 1300		
18–8	N42W, 62NE	N100	N82W, 26SW			0, 90	−25	660	250, 13000		
							(160)		1300, 1900		

Sep.: Separation measured in the direction of fault trace on the outcrop.
R: Reverse separation.
N: Normal separation.
*: Slip direction rake, calculated from apparent conjugate relationships.
Rake greater than 180° is equivalent to that with 360° subtracted from it.
Separation in parentheses is estimated.

the geometry of the fault. When the trace direction of the marker plane and the striation direction lie close to each other on the fault surface, small errors could reverse the calculated slip direction (Fig. 3g). The comparison of slip directions with maximum shear stress direction on the fault derived from estimated palaeostresses commonly shows only one of the two senses of slip to be compatible with the directions of maximum shear stress. When the reversal of the calculated slip does not occur (Fig. 3f), the sense of slip is determined by the calculation. Because this method assumes only a small range of errors for the slip from the observed striation direction, if the fault slips are large and slip directions were altered greatly during the fault development, the calculated magnitude of slip will be unreliable and so may be even the sense of slip. Therefore, the sense of slip determined should be checked, if possible, with that determined on the basis of the character of fault surface or secondary shears.

Acknowledgements—We would like to express hearty thanks to Dr R. J. Lisle, Dr T. G. Blenkinsop and Dr M. F. Howells for many valuable suggestions for improvement of the original manuscript.

REFERENCES

Billings, M. P. 1954. *Structural Geology* (2nd edition). Prentice-Hall Inc., Englewood Cliffs, New Jersey.

Ragan, D. M. 1973. *Structural Geology* (2nd edition). Wiley & Sons, New York.

Yamada, E. 1980. Computer program to calculate the net slip of faults and the separation of planes by the faults. *Bull. geol. Surv. Japan* **31**, 567–584.

Yamada, E. & Sakaguchi, K. 1991. Revised method to compute principal stress directions and *R*-values from striations on fracture planes. *Bull. geol. Surv. Japan* **42**, 503–515.